

# Real Options and Signaling in Strategic Investment Games

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## Abstract

A game with an incumbent and an entrant is considered. Each of them decides a timing of an investment into a new project. The profit flows of both firms involve two uncertain factors: (i) demand for the market observed only by the incumbent, and (2) fluctuation of the profit flow described by a geometric Brownian motion observed by both firms. The optimal timing of the investment of the incumbent under high demand is earlier than that under the low demand, if the entrant's timing is fixed. However, the earlier investment of the incumbent may reveal information of high demand to the entrant, so that the early investment of the incumbent would accelerate the timing of the investment of the entrant. This earlier investment of the entrant reduces monopolistic profit of the incumbent before the investment of the entrant. Therefore, the incumbent under high demand may delay the timing of the investment strategically in order to hide the information. In the present paper, equilibria of this signaling game are characterized and sufficient conditions for the strategic behavior of the incumbent are obtained. The values of both firms in equilibria are compared with the case of complete information and the welfare loss is investigated.

**Keywords:** game theory, real option, investment timing, asymmetric information

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# 1 Introduction

Timings of the investment of firms are affected by the uncertainty. In contrast to the traditional net present value (NPV) model, the concept of real options clarifies the nature of the strategic delay of the irreversible investment under uncertainty. Previous studies, for example, [1] and [14], assert that a firm should wait for an investment even if the net present value is positive and the optimal timing of the investment is delayed beyond the traditional Marshallian threshold. This concept has been developed into the real option approach, summarized by [2].

On the other hand, the timings of investments are also affected by market competition. Thus, the stream of research for real option has recently been extended to investments under competition by combining real option approach with game theory. A typical model incorporating real option approach into game theory is sometimes referred to as an *investment game*, in which two firms decide the timings of option exercises in a duopolistic market. Previous studies, such as [19], [3], [12], [9], and [20], investigated competition by symmetric firms. An important implication about the previous studies is that an advantage of the first mover and a negative externality of the rival's investment reduce the value for the options of the firms and accelerate the timing of the investment. [16] and [11] obtained the results for two asymmetric firms, but the information for the two firms was assumed to be identical.

Asymmetry of information in an investment game also influences the timing of the exercise. [13] modeled an investment game with two competitive firms under incomplete information, in which the investment cost of each firm is private information. [7] consider the situation in which one firm has complete information about the investment cost of its rival, whereas the rival firm has incomplete information about the investment cost of the first firm.

These studies examined investment games based on asymmetric information in which the options exercised by one firm do not influence information of the other firm. However, under

asymmetric information, a behavior of a firm acting earlier may reveal information to the other firms acting later. Hence, the firm who invests earlier considers strategic exercise of the option to hide information. This strategic behavior sometimes conflicts with the optimal timing of the exercise.

In the present paper, influence of this strategic transmission of information called signaling on investments is examined under uncertainty and competition. I attempt this analysis by the most typical but simplest model of real option and game theory. I consider a model of an investment game with two asymmetric firms, an incumbent and an entrant, who have the option to invest in a new project. The profit flow of each firm has two uncertainty factors. One factor is the potential size of the market of the investment, which is referred to as the level of demand that has been determined at the beginning of the game. The level of demand is assumed to take one of two possible values, i.e., high or low. The level of demand can be observed only by the incumbent as private information due to some experiences of the incumbent, whereas the entrant cannot obtain the information. The other factor is fluctuation of the profit flow given by a stochastic process that is common to both firms. In my framework, the incumbent invests earlier than the entrant for any level of the demand because the market share of the incumbent is assumed to be sufficiently larger and the investment cost of the incumbent is assumed to be sufficiently smaller than the entrant. If the incumbent considers the optimal timing of the investment under a fixed timing of the entrant, information of the level of the demand would be revealed to the entrant by an observation of the incumbent's timing. Then, the entrant observing an earlier investment of the incumbent would accelerate the timing of the investment. Since this accelerated timing of the entrant's investment would reduce monopolistic period for the incumbent and reduce the incumbent's profit. Hence, the incumbent may strategically delay the timing of the investment to hide information.

Under this setting, the present study investigates the following three questions: (i) what conditions cause this strategic behavior of the incumbent?; (ii) which factors influence this strategic behavior of the incumbent? and how these factors affect the value of the both firms?; and (iii) how are the values of the firms affected in the presence of asymmetric information as compared to complete information. About these questions, the present paper obtains the following answers.

With regard to question (i), since the incumbent under low demand does not have an incentive to mimic the timing of the incumbent under high demand, only the incumbent under high demand has an incentive to behave strategically by delaying the timing. Solving a threshold in which the incumbent under high demand delays the investment strategically, conditions for this strategic behavior are obtained. In addition, it is also shown that there exists a certain range of the parameters in which no pure strategy equilibrium exists and there exists a mixed strategy equilibrium. The probability of the earlier investment for the high-demand incumbent in an equilibrium with mixed strategies is also identified.

With regard to question (ii), we show that an initial condition of fluctuation of profit flow does not affect strategic behavior of the incumbent. The strategic behavior of the incumbent depends on marginal profits of both firms. In particular, the smaller marginal profit in duopoly of the high-demand incumbent causes the incumbent to act strategically. Similarly, the smaller investment cost of the incumbent cause strategic behavior of the high-demand incumbent. The larger investment cost of the entrant also cause strategic behavior of the high-demand incumbent. The above results are obtained analytically, but I cannot obtain the effect of volatility. I show a numerical example that the larger volatility causes the strategic behavior of the high-demand incumbent.

With regard to question (iii), for parameters in which the incumbent under high demand

does not behave strategically, all players, the entrant, the high-demand incumbent and the low-demand incumbent, have no loss or no gain as compared to the case of complete information. In contrast, for parameters in which the incumbent under high demand behave strategically, the high-demand incumbent increases the values so as to mimic the low-demand incumbent, whereas the low-demand incumbent decreases the values as compared to the case of complete information. The values of the entrant under both levels of demand decrease by distorting the optimal timing of the exercise of the option because the entrant cannot distinguish the precise demand and exercises the optimal timing of the investment under the expected level of demand.

Under a mixed strategy equilibrium, it is shown that the ex ante value of the high-demand incumbent is identical to that of complete information, whereas the values of the low-demand incumbent and the entrant decrease.

I have to note the related literature about the strategic behavior under the presence of asymmetric information in the framework of real option. [6] investigated conflicts between managers and owners and presented a model of the investment timing by managers by combining real options with contract theory. [17] and [18] also examined manager-shareholder conflicts arising from asymmetric information in the context of the real option approach. [15] investigated a signaling game between an informed firm and an outside investor, in which the firm decides both the timing of investment and the debt-equity mix. In [15], the presence of asymmetric information and the signaling effect erode the option value of the firm. [5] investigated a similar model that considers the conflicts between continuum types of an informed agent and an outsider. Information revelation involving several firms was investigated by [4], where each firm has private information about the payoff uncertainty and updates the belief for the payoff by observing the strategies exercised by other firms. [4] focused on informational cascades and projects in which firms are not in competition with each other. Thus, the firms reveal their private information

truthfully.

Although each model mentioned above may regard a signaling game of real options, the present study considers a different situation in which competitive firms in a duopoly market play a signaling game for their investments. In my knowledge this is the first study considering signaling effects in this framework of real option.

The remainder of the present paper is organized as follows. Section 2 describes the notation used herein and presents a description of the model. Section 3 presents the value of the entrant and non-strategic values of the incumbents, which implies a benchmark of the analysis. In Section 4, two candidate solutions, either of which is a separating equilibrium or a pooling equilibrium, respectively, are defined. Conditions that either of the two candidate becomes an equilibrium are also presented. Although these conditions characterize an equilibrium in pure strategies, in some cases, there is no equilibrium in pure strategies. Section 5 deals with mixed strategies and presents the conditions of the equilibria. Since these equilibria in mixed strategies include the case of equilibria in pure strategies examined in the previous section, the conditions characterize the equilibrium comprehensively. In Section 6, the manner in which the values of firms are affected by the presence of asymmetric information is examined. The gains and losses of the values for both high-demand and low-demand incumbents and the entrant are compared with the case of complete information. The conditions of the manipulative revelation for the duopoly profit and the costs of the incumbent and the entrant are also examined. Section 7 presents numerical examples, and Section 8 presents conclusions and discusses future research.

## **2 The Model**

Two asymmetric firms, an incumbent and an entrant, each of which has the option to the investment into a new project are considered. The incumbent and the entrant are denoted as

firm  $I$  and firm  $E$ , respectively. The investments of the projects are assumed to be irreversible, and the sunk cost of the investment of firm  $i$  is denoted as  $K_i$  for  $i = I, E$ . It is assumed that revenue flow from the investment can be obtained without time lag. Profits by the investments are obtained from the same market, and hence the profit of each firm is decreasing by the investment of the rival firm and depends on common uncertain factors stated as follows.

One uncertain factor of profits represents a stochastic process, denoted by  $X_s$ , as a standard real option setting. Here,  $X_s$  is interpreted as unsystematic shocks of the demand over time and is common to both firms.

Suppose that  $X_s$  follows a geometric Brownian motion:

$$dX_s = \mu X_s dt + \sigma X_s dz$$

where  $\mu$  is the drift parameter,  $\sigma$  is the volatility parameter, and  $dz$  is the increment of a standard Wiener process. Both firms are assumed to be risk neutral, with risk free rate of interest  $r$ . Finally,  $r > \mu$  is assumed for convergence.

The other uncertain factor of the profit represents a systematic risk and is assumed to be constant over time. This factor is denoted by  $\theta$ , where  $\theta = H$  and  $\theta = L$  mean that the basic level of demand is high and low, respectively. The prior probabilities of  $\theta = H$  and  $\theta = L$  are denoted as  $p$  and  $1 - p$ , respectively.

When only one firm invests in a project, profit flow of firm  $i$  under the monopoly becomes  $\pi_{i1}^\theta X_s$ . On the other hand, the profit flow of firm  $i$  under the duopoly in which both firms have already invested in each project becomes  $\pi_{i2}^\theta X_s$ . The profit flow of any firm that has not invested yet is assumed to be zero.  $\pi_{i1}^\theta$  and  $\pi_{i2}^\theta$  is called a marginal profit of firm  $i$  (the incumbent or the entrant) under demand level  $\theta$  in monopoly and duopoly, respectively.  $\pi_{i1}^\theta > \pi_{i2}^\theta > 0$ , is assumed for any  $i = I, E$  and  $\theta = H, L$ , i.e., the marginal profit in monopoly is always greater than that in duopoly for each firm and each level of demand at the same  $X_s$ . Moreover,  $\pi_{ij}^H > \pi_{ij}^L > 0$  is

assumed for  $i = I, E$  and  $\theta = H, L$ , i.e., a marginal profit under high demand is always greater than that under low demand.

The incumbent has several advantages over the entrant due to experiences that the incumbent gained through past activities. The incumbent has more information, a greater share of the products, and a lower investment cost than the entrant. In detail, the incumbent has the following two advantages. First, while  $X_s$  is observable by two firms, the uncertain factor  $\theta$  can be observed only by the incumbent, i.e.,  $\theta$  is the private information of the incumbent. Second,  $K_I/\pi_{I2}^L$  is assumed to be smaller than  $K_E/\pi_{E1}^H$ . This assumption holds if the marginal profit in monopoly of the incumbent is sufficiently larger than that of the entrant and/or the cost of the investment  $K_I$  is sufficiently smaller than  $K_E$ .

### 3 Value Functions of a Benchmark Case

The proposed model is one of an option exercise game that is investigated under the joint framework of real options and game theory. A number of studies, including [19], [3], [10], [12], [9], [8], and [20], have considered symmetric firms in order to examine the preemptive behavior of competition. In these models, if the value of the optimal entry of the leader is greater than the value of the entry for the best reply of the follower, then both firms want to become a leader. In this case, the optimal threshold of the leader is obtained solving a system of equations of equilibria, and the value of the leader is not determined by maximizing the expected profit of either firm. [8], [11] and [16] demonstrated that this preemptive behavior and simultaneous entry would occur under asymmetry of costs and profits. In this case, obtaining the equilibrium values is complicated.

However, if the asymmetry is sufficiently large and the initial value of both firms are sufficiently small to wait for the investment, the lower-cost firm must be the leader, (see [11] and



[16]). Based on the results of [11], two assumptions in the present model, i.e.,  $K_I/\pi_{I2}^L > K_E/\pi_{E1}^H$  and sufficiently small  $X_t = x$ , imply that the incumbent must be the leader and that the entrant must be the follower.

Due to this setting, the decisions and the values of both firms can be analyzed under the condition in which the incumbent is the leader and the entrant is the follower.

### 3.1 Value of the Entrant

The benchmark case is solved by backward induction. The value of the entrant is solved first and the value of the incumbent as the leader is discussed later because the incumbent is the leader and the entrant is the follower,

The entrant does not know the level of the demand certainly, so that the entrant invests at the optimal timing based on the belief of the level of the demand. Let  $u_E^*(q)$  be the value of the entrant under the condition that the entrant believes that high demand will occur with probability  $q$ .

$u_E^*(q)$  is given by

$$u_E^*(q) = \max_{t_E} E\left[\int_{t_E}^{\infty} e^{-r(s-t)}(q\pi_{E2}^H + (1-q)\pi_{E2}^L)X_s ds - e^{-r(t_E-t)}K_E | X_t = x\right].$$

In this setting, a threshold strategy is sufficient to give the optimal stopping time. Hence the problem is written by deciding the optimal threshold  $x_E$ , as follows:

$$u_E^*(q) = \max_{x_E} E\left[\int_{\tau(x_E)}^{\infty} e^{-r(s-t)}(q\pi_{E2}^H + (1-q)\pi_{E2}^L)X_s ds - e^{-r(\tau(x_E)-t)}K_E | X_t = x\right].$$

where  $\tau(\tilde{x})$  denote the first hitting time at threshold  $\tilde{x}$ , i.e.,  $\tau(\tilde{x}) = \inf\{s \geq t | X_s \geq \tilde{x}\}$ . Let  $x_E^*(q)$  be the optimal threshold for belief  $q$ . The usual calculation of real option analysis (e.g., [2]) implies that

$$x_E^*(q) = \frac{\beta}{\beta - 1} \frac{r - \mu}{q\pi_{E2}^H + (1-q)\pi_{E2}^L} K_E \quad (1)$$

where  $\beta$  is defined by

$$\beta = \frac{1}{2} - \frac{\mu}{\sigma^2} + \sqrt{\left(\frac{1}{2} - \frac{\mu}{\sigma^2}\right)^2 + \frac{2r}{\sigma^2}}. \quad (2)$$

Let  $x_E^H = x_E^*(1)$ ,  $x_E^L = x_E^*(0)$ , and  $x_E^M = x_E^*(p)$ . Here,  $x_E^H$  and  $x_E^L$  are the thresholds when the entrant believes that the demands are high and low, respectively. In addition,  $x_E^M$  is the threshold when the entrant predicts high demand with prior probability  $p$ .

We can verify that

$$x_E^H \leq x_E^M \leq x_E^L, \quad (3)$$

because  $\pi_{E2}^H \geq \pi_{E2}^L$ .

### 3.2 Value of the Incumbent

Since the incumbent taking into account the strategic exercise chooses the timing of the investment that may not be optimal under a fixed timing of the entrant, the value of the incumbent can be expressed by a function of a threshold of the timing of the investment chosen by the incumbent. The value of the incumbent also depends on the level of the demand that can be observed by the incumbent, and on the timing of the entrant. Let  $u_I(x_I, x_E, \theta)$  be the expected profit of the incumbent under  $\theta$ , when the incumbent invests at the threshold  $x_I$  and the entrant invests at  $x_E$ . Note that  $x_I < x_E$ .

Here,  $u_I(x_I, x_E, \theta)$  is given by

$$u_I(x_I, x_E, \theta) = E\left[\int_{\tau(x_I)}^{\tau(x_E)} e^{-r(s-t)} \pi_{I1}^\theta X_s ds - e^{-r(\tau(x_I)-t)} K_I + \int_{\tau(x_E)}^{\infty} e^{-r(s-t)} \pi_{I2}^\theta X_s ds \mid X_t = x\right].$$

$u_I(x_I, x_E, \theta)$  can be rewritten as

$$u_I(x_I, x_E, \theta) = v_I(x_I, \theta) - w_I(x_E, \theta),$$

where

$$v_I(x_I, \theta) = E\left[\int_{\tau(x_I)}^{\infty} e^{-r(s-t)} \pi_{I1}^\theta X_s ds - e^{-r(\tau(x_I)-t)} K_I \mid X_t = x\right],$$

and

$$w_I(x_E, \theta) = E\left[\int_{\tau(x_E)}^{\infty} e^{-r(s-t)}(\pi_{I1}^{\theta} - \pi_{I2}^{\theta})X_s ds | X_t = x\right].$$

In the following, in order to simplify the analysis, the initial condition  $x$  is assumed to be sufficiently small, indicating that the incumbent for any demand has not yet invested at the initial time. Hence, only the case in which  $x \leq x_I$  is examined. Since  $x_I < x_E$ ,  $x$  is also less than  $x_E$ . Under these assumptions,  $v_I(x_I, \theta)$  and  $w_I(x_E, \theta)$  are expressed as the following proposition, which can be derived by the strong Markov property of the geometric Brownian motion and the calculation for the hitting time.

**Proposition 3.1.**  $u_I(x_I, x_E, \theta)$  is given by

$$u_I(x_I, x_E, \theta) = v_I(x_I, \theta) - w_I(x_E, \theta). \quad (4)$$

where

$$v_I(x_I, \theta) = \left(\frac{\pi_{I1}^{\theta}}{r-\mu}x_I - K_I\right) \left(\frac{x}{x_I}\right)^{\beta} \quad x \leq x_I \quad (5)$$

and

$$w_I(x_E, \theta) = \frac{\pi_{I1}^{\theta} - \pi_{I2}^{\theta}}{r-\mu} x_E \left(\frac{x}{x_E}\right)^{\beta}, \quad x < x_E. \quad (6)$$

**Proof.** See Appendix.

Note that (3) yields

$$w_I(x_E^H, \theta) \geq w_I(x_E^M, \theta) \geq w_I(x_E^L, \theta) \quad (7)$$

because  $w_I(x_E, \theta)$  is a decrement in the threshold  $x_E$ .

If  $x_E$  is independent of the incumbent decision  $x_I$ , then  $w_I(x_E, \theta)$  does not depend on  $x_I$ . Then, the incumbent can maximize the expected profit only by  $v_I(x_I, \theta)$  when  $x_I$  is given.

$x_E$  in a signaling game, which is examined in the next section, depends on  $x_I$ . In the remainder of this section, however, the case in which  $x_E$  is independent of  $x_I$  is examined as a

benchmark of the analysis. Let  $x_I^\theta$  be the optimal threshold of the incumbent under demand  $\theta$  when  $x_E$  is given independent of  $x_I$ . Then,  $v_I(x_I^\theta, \theta)$  is expressed by

$$v_I(x_I^\theta, \theta) = \max_{x_I} v_I(x_I, \theta) = \max_{t_I} E^x \left[ \int_{t_I}^{\infty} e^{-r(s-t)} \pi_{I1}^\theta X_s ds - e^{-r(s-t_I)} K_E \right]. \quad (8)$$

Standard calculation of the real option approach<sup>1</sup> implies that

$$x_I^\theta = \frac{\beta}{\beta - 1} \frac{r - \mu}{\pi_{I1}^\theta} K_I, \quad (9)$$

and

$$v_I(x_I^\theta, \theta) = \begin{cases} \frac{K_I}{\beta - 1} \left( \frac{x}{x_I^*(\theta)} \right)^\beta & x \leq x_I^\theta, \\ \frac{\pi_{I1}^\theta}{r - \mu} x - K_I & x > x_I^\theta. \end{cases}$$

## 4 Equilibrium in Pure Strategies

### 4.1 Definitions of the Solution

In the following, I define a concept of an equilibrium. A weak perfect Bayesian equilibrium is applied as the solution concept.

A three tuples  $\{(a_I(H), a_I(L)), a_E(\cdot), q(\cdot)\}$  is said to be an *assessment* where

- $a_I(H)$  and  $a_I(L)$  are the threshold of the incumbent for  $H$  and  $L$ , respectively,
- $a_E(x_I)$  is the threshold of the entrant for the threshold  $x_I$  of the observed incumbent, and
- $q(x_I)$  is the belief of the entrant for the threshold  $x_I$  of the observed incumbent.

In this section, Section 4, only pure strategies is concerned to describe results briefly, whereas I extend the results to mixed strategies in next section, Section 5.

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<sup>1</sup>Both  $x_I^*(\theta)$  and  $v_I(x_I^*(\theta), \theta)$  are calculated based on the smooth pasting condition and the value matching condition of real option approach. These conditions can also be derived from the first-order condition to maximize  $v_I(x_I, \theta)$ , which is obtained by differentiating (5).

An assessment  $\{(a_I^*(H), a_I^*(L)), a_E^*(\cdot), q^*(\cdot)\}$  is said to be an equilibrium if the assessment satisfies if the following three conditions.

First, for  $\theta = H, L$ ,  $a_I^*(\theta)$  is the optimal threshold of the incumbent;

$$u_I(a_I^*(\theta), a_E^*(a_I^*(\theta)), \theta) = \max_{x_I} u_I(x_I, a_E^*(x_I), \theta). \quad (10)$$

Second, for any observing the entry of the incumbent at  $x_I$ ,  $a_E^*(x_I)$  is the optimal threshold of the entrant under belief  $q^*(x_I)$ ;

$$a_E^*(x_I) = x_E^*(q^*(x_I)). \quad (11)$$

Third, the belief of the entrant  $q^*(x_I)$  is consistent with the equilibrium strategy of the incumbent  $(a_I^*(H), a_I^*(L))$  in the sense of Bayes rule. The consistent belief  $q^*(x_I)$  is calculated as follows. Recall that  $q^*(x_I)$  is defined as  $Prob[\theta = H|x_I]$ . and by Bayes' rule implies that  $Prob[\theta = H|x_I]$  is calculated as

$$Prob[\theta = H|x_I] = \frac{Prob[x_I|\theta = H]Prob[\theta = H]}{Prob[\theta = H]Prob[x_I|\theta = H] + Prob[\theta = L]Prob[x_I|\theta = L]}.$$

Substituting  $Prob[\theta = H] = p$  and  $Prob[\theta = L] = 1 - p$ , the consistent belief is expressed by

$$q^*(x_I) = \frac{pProb[x_I|\theta = H]}{pProb[x_I|\theta = H] + (1 - p)Prob[x_I|\theta = L]}. \quad (12)$$

As I mentioned above, I only consider pure strategies while mixed strategies are investigated in Section 5. Hence,  $Prob[x_I|\theta = H]$  and  $Prob[x_I|\theta = L]$  can be explicitly written as

$$Prob[x_I|\theta = H] = \begin{cases} 1 & a_I^*(H) = x_I \\ 0 & a_I^*(H) \neq x_I, \end{cases} \quad Prob[x_I|\theta = L] = \begin{cases} 1 & a_I^*(L) = x_I \\ 0 & a_I^*(L) \neq x_I. \end{cases} \quad (13)$$

Equations (12) and (13) imply that

$$q^*(x_I) = \begin{cases} p & x_I = a_I^*(H) \text{ and } x_I = a_I^*(L), \\ 1 & x_I = a_I^*(H) \text{ and } x_I \neq a_I^*(L), \\ 0 & x_I \neq a_I^*(H) \text{ and } x_I = a_I^*(L). \end{cases} \quad (14)$$

Note that if  $a_I^*(H) \neq x_I$  and  $a_I^*(L) \neq x_I$ , then any belief  $q^*(x_I)$  is consistent.

Thus, in pure strategies, a perfect Bayesian equilibrium is formally defined as follows.

**Definition 4.1.** *An assessment is said to be a perfect Bayesian equilibrium in pure strategies if the assessment satisfies (10), (11), and (14).*

A perfect Bayesian equilibrium in pure strategies is said to be a *pooling* equilibrium if  $a_I^*(H) = a_I^*(L)$ . (14) implies that

$$q^*(a_I^*(H)) = q^*(a_I^*(L)) = p.$$

A pooling equilibrium corresponds to the case in which the actions of the incumbent do not convey information about the demand, and the entrant predicts high demand with prior probability  $p$ . Therefore, in the pooling equilibrium, the threshold of the entrant in the equilibrium is

$$a_E^*(a_I^*(H)) = a_E^*(a_I^*(L)) = x_I^M$$

because  $x_I^M = x_I^*(p)$ .

A perfect Bayesian equilibrium in pure strategies is said to be a *separating* equilibrium if  $a_I^*(H) \neq a_I^*(L)$ . In the separating equilibrium, (14) implies that

$$q^*(a_I^*(H)) = 1, \quad q^*(a_I^*(L)) = 0.$$

This means that the entrant determines the level of the demand exactly by observing the actions of the incumbent. Hence, the threshold of the entrant in the separating equilibrium is

$$a_E^*(a_I^*(H)) = x_E^H, \quad a_E^*(a_I^*(L)) = x_E^L.$$

## 4.2 Candidates of the Outcome

The incumbent exactly invest at  $x_I^H$  or  $x_I^L$  if the entrant invests at fixed timing. However, timing of the incumbent's investment cannot be limited to  $x_I^H$  and  $x_I^L$ , and I have to set the

entrant's belief at any timing not restricted to  $x_I^H$  and  $x_I^L$  in an equilibrium. This cause the many possibility of equilibria. Such an equilibrium may be chosen by the theory of equilibrium refinement in game theory, but it makes the analysis difficult.

In the present model, only the incumbent under high demand has an incentive of mimicking incumbent under low demand while the incumbent under low demand does not have an incentive of mimicking and wants to inform this type truthfully to the entrant. Hence, I assume that if the entrant observes the unexpected timing of the investment, neither  $x_I^H$  nor  $x_I^L$ , the entrant believes that it is a strategic behavior under the high demand.

By this consideration, I focus on the following two types of equilibrium for the candidate of the outcome. The first type is a separating equilibrium satisfying that

$$\begin{aligned}
 a_I^*(H) &= x_I^H, & a_I^*(L) &= x_I^L \\
 a_E^*(x_I) &= \begin{cases} x_E^H & x_I \neq x_I^L, \\ x_E^L & x_I = x_I^L, \end{cases} \\
 q^*(x_I) &= \begin{cases} 1 & x_I \neq x_I^L, \\ 0 & x_I = x_I^L. \end{cases}
 \end{aligned} \tag{15}$$

In the separating equilibrium (15), the incumbent for any demand truthfully invests at the optimal threshold with respect to the demand. This truthful behavior reveals the information of the demand that the incumbent possesses. The entrant obtains the information about the demand by observing the behavior of the incumbent and invests optimally with complete information. If the entrant observes that the incumbent invests at neither  $x_I^H$  nor  $x_I^L$ , then any belief of the entrant is consistent. In other words, the belief of the entrant is assigned arbitrarily in the observation of the entrant in this off-equilibrium path.

The second type of equilibria is a pooling equilibrium satisfying that

$$\begin{aligned}
 a_I^*(H) &= a_I^*(L) = x_I^L \\
 a_E^*(x_I) &= \begin{cases} x_E^H & x_I \neq x_I^L, \\ x_E^M & x_I = x_I^L, \end{cases} \\
 q^*(x_I) &= \begin{cases} 1 & x_I \neq x_I^L, \\ p & x_I = x_I^L \end{cases}
 \end{aligned} \tag{16}$$

In the pooling equilibrium (16), the high-demand incumbent does not invest at the optimal threshold of the high demand under the fixed timing of the entrant, but rather invests at the threshold of the low demand under the fixed timing of the entrant. This delay of the investment hides the information about the high demand, and the entrant cannot distinguish the demand by observing the behavior of the incumbent. Thus, the entrant predicts the level of the demand according to the prior probability and invests at the threshold for the expectation of the demand. The entrant is assumed to believe that high demand occurs in the off-equilibrium path, as well as the separating equilibrium (15).

### 4.3 Conditions for an Equilibrium

In this subsection, I investigate necessary and sufficient conditions in which each of separating equilibrium (15) or pooling equilibrium (16) exists. Since both (15) and (16) satisfies (11) and (14) by the definition, it remains to consider (10), the optimality of the incumbent, under the strategy  $a_E^*(\cdot)$  and belief  $q^*(\cdot)$ . In other words, (10) implies the necessary and sufficient conditions. Moreover, the incumbent under low demand does not have an incentive to deviate from  $x_I^L$  because pretending the high-demand incumbent only accelerates the timing of the entrant's investment and reduces the value of the incumbent. Hence, only the timing of the incumbent under the high demand decides equilibrium conditions.



First, assume that (15) is an equilibrium. In (15), the entrant observing the investment of the incumbent at  $x_I^\theta$  for  $\theta = H, L$  invests at  $x_E^\theta$ . If (15) is an equilibrium, then the deviation from  $x_I^H$  to  $x_I^L$  does not increase the payoff of high-demand type incumbent. Hence, the following condition should hold:

$$u_I(x_I^H, x_E^H, H) \geq u_I(x_I^L, x_E^L, H). \quad (17)$$

Second, assume that (16) is an equilibrium. In (16), both types of the incumbent invest at  $x_I^L$ . The entrant observing the incumbent's investment at  $x_I^L$  predicts the level of the demand by prior probability  $p$ , so that the expectation of the profit flow is  $\pi_{E2}^M$ . The entrant then invests at  $x_E^M$ , which is optimal for  $\pi_{E2}^M$ . Note that the entrant believes the high demand in (16) when the entrant observes the investment at  $x_I^H$ . If (16) is an equilibrium, the incumbent under the high demand does not have an incentive to deviate from  $x_I^L$  to  $x_I^H$ , and hence the following condition should hold:

$$u_I(x_I^H, x_E^H, H) \leq u_I(x_I^L, x_E^M, H). \quad (18)$$

(17) and (18) are not only necessary conditions. The following proposition asserts that (17) and (18) are also sufficient conditions of the equilibrium.

**Proposition 4.2.** (a) (17) holds if and only if separating equilibrium (15) exists.

(b) (18) holds if and only if pooling equilibrium (16) exists.

*Proof.* First, it is shown that if (17) holds,  $\{(a_I^*(H), a_I^*(L)), a_E^*(\cdot), q^*(\cdot)\}$  satisfying (15) is a perfect Bayesian equilibrium in pure strategies. In order to prove this relation, it is sufficient to show that the assessment satisfies three conditions, (10), (11), and (14). (17) has been already satisfied (11) and (14). It remains to show that (15) satisfies (10):

$$u_I(a_I^*(\theta), a_E^*(a_I^*(\theta)), \theta) \geq u_I(x_I, a_E^*(x_I), \theta). \quad (19)$$

and for any  $x_I \neq a_I^*(\theta)$ .

Let  $\theta = L$ . Since, in (15),  $a_I^*(L) = x_I^L$ ,  $a_E^*(x_I^L) = x_E^L$ , and  $a_E^*(x_I) = x_E^H$  for any  $x_I \neq x_I^L$ , we need only show that  $u_I(x_I^L, x_E^L, L) \geq u_I(x_I, x_E^H, L)$  for any  $x_I \neq x_I^L$ . As  $v_I(x_I^L, L) \geq v_I(x_I, L)$  by (8) and (7) yields  $w_I(x_E^H, L) \geq w_I(x_E^L, L)$ ,

$$u_I(x_I^L, x_E^L, L) = v_I(x_I^L, L) - w(x_E^L, L) \geq v_I(x_I, L) - w_I(x_E^H, L) = u_I(x_I, x_E^H, L).$$

Hence, (19) holds for  $\theta = L$ .

Second, let  $\theta = H$ . Then, I have to prove  $u_I(x_I^H, x_E^H, H) \geq u_I(x_I, x_E^H, H)$  for any  $x_I \neq x_I^L$  and  $u_I(x_I^H, x_E^H, H) \geq u_I(x_I^L, x_E^L, H)$ . As  $v_I(x_I^H, H) \geq v_I(x_I, H)$  for any  $x_I$  by (8),

$$u_I(x_I^H, x_E^H, H) = v_I(x_I^H, H) - w(x_E^H, H) \geq v_I(x_I, H) - w_I(x_E^H, H) = u_I(x_I, x_E^H, H).$$

$u_I(x_I^H, x_E^H, H) \geq u_I(x_I^L, x_E^L, H)$  holds because of (17). Then, a separating equilibrium (15) exists.

Conversely, it is herein proven that if (15) is a perfect Bayesian equilibrium in pure strategies, then (17) holds. Otherwise, assume that  $u_I(x_I^H, x_E^H, H) < u_I(x_I^L, x_E^L, H)$ . Then, the high-demand incumbent strictly increases the payoff by deviating  $x_I^L$  from  $a_I^*(H) = x_I^H$ , which means that (15) is not an equilibrium. This completes the proof of (a).

The proof of (b) is obtained in a similar manner.

□

Note that neither (15) nor (16) is a perfect Bayesian equilibrium in pure strategies for  $u_I(x_I^L, x_E^M, H) < u_I(x_I^H, x_E^H, H) < u_I(x_I^L, x_E^M, H)$  because  $u_I(x_I^L, x_E^M, H) \leq u_I(x_I^L, x_E^L, H)$ . Under this condition, the mixed strategy of the incumbent should be considered in order to ensure the existence of the equilibrium, investigated in Section 5.

## 5 Equilibria in Mixed Strategies

In order to examine mixed strategies of the incumbent, I introduce the following notation. Let  $x_I(\lambda)$  be a mixed action of the incumbent, where the incumbent chooses  $x_I^H$  with probability  $\lambda$  and  $x_I^L$  with probability  $1 - \lambda$  for  $0 \leq \lambda \leq 1$ . Note that even if the incumbent uses the mixed action, the entrant observes only a realized action, either  $x_I^H$  or  $x_I^L$  in the equilibrium. Hence, the entrant takes either  $a_E(x_I^H)$  or  $a_E(x_I^L)$  in the equilibrium.  $u_I$  is extended to the set of mixed actions  $x_I(\lambda)$  for  $0 \leq \lambda \leq 1$ , as defined by

$$u_I(x_I(\lambda), a_E(\cdot), \theta) = \lambda u_I(x_I^H, a_E(x_I^H), \theta) + (1 - \lambda) u_I(x_I^L, a_E(x_I^L), \theta)$$

for any  $x_E$  and  $\theta = H, L$ . Here,  $u_I(x_I(\lambda), a_E(\cdot), \theta)$  denotes the expected payoff of the incumbent, where the incumbent uses mixed action  $x_I(\lambda)$ , and the entrant follows  $a_E(\cdot)$ .

As the analysis in Section 4, we restricted the candidates of the outcome to  $a_I^*(H) = x_I(\lambda)$  and  $a_I^*(L) = x_I^L$  because the low-demand incumbent in the equilibrium does not have an incentive to deviate from  $x_I^L$ . The consistent belief of the entrant  $q^*(\cdot)$  for  $a_I^*(H) = x_I(\lambda)$  and  $a_I^*(L) = x_I^L$  is derived by Bayes rule, given by (12). Here,  $Prob[x_I|\theta = H]$  and  $Prob[x_I|\theta = L]$  are given by

$$Prob[x_I|\theta = H] = \begin{cases} \lambda & x_I = x_I^H \\ 1 - \lambda & x_I = x_I^L \\ 0 & x_I \neq x_I^H, x_I^L, \end{cases} \quad Prob[x_I|\theta = L] = \begin{cases} 1 & x_I = x_I^L \\ 0 & x_I \neq x_I^L, \end{cases} \quad (20)$$

(12) and (20) imply the consistent belief  $q^*(\cdot)$ , as follows:

$$q^*(x_I^H) = \frac{p\lambda}{p\lambda + (1-p) \times 0} = 1$$

and

$$q^*(x_I^L) = \frac{p(1-\lambda)}{p(1-\lambda) + (1-p) \times 1} = \frac{p(1-\lambda)}{1-p\lambda}.$$

The consistent belief  $q^*(\cdot)$  indicates that the entrant observing  $x_I^H$  completely learns the high demand with probability one, because only the incumbent with information of the high demand

invests at  $x_I^H$ . Hence, the optimal timing of investment of the entrant observing the incumbent's investment at  $x_I^H$  is  $x_E^H$ . On the other hand, since both types of the incumbents have positive probabilities of the investment at  $x_I^L$ , the entrant observing that the incumbent acted at  $x_I^L$  predicts the high demand according to probability  $q^*(x_I^L)$ . The optimal timing of the entrant's investment observing  $x_I^L$  is  $x_E^*(q^*(x_I^L))$ . For simplicity, let  $x_E^*(q^*(x_I^L))$  be  $x_E^\lambda$ .

Now we consider a hybrid equilibrium satisfying that

$$\begin{aligned} a_I^*(H) &= x_I(\lambda), & a_I^*(L) &= x_I^L, \\ a_E^*(x_I) &= \begin{cases} x_E^H & x_I \neq x_I^L \\ x_E^\lambda & x_I = x_I^L, \end{cases} \\ q^*(x_I) &= \begin{cases} 1 & x_I \neq x_I^L \\ \frac{p(1-\lambda)}{1-p\lambda} & x_I = x_I^L \end{cases} \end{aligned} \quad (21)$$

for  $0 \leq \lambda \leq 1$ .

Note that a hybrid equilibrium (21) for  $\lambda = 1$  and  $\lambda = 0$  are identical to (15) and (16), respectively. Hence, an equilibrium condition for a hybrid equilibrium (21) includes conditions for (15) and (15) comprehensively.

Let  $\{(a_I^*(H), a_I^*(L)), a_E^*(\cdot), q^*(\cdot)\}$  be a hybrid equilibrium (21) for  $0 \leq \lambda \leq 1$ . Then the high-demand incumbent does not have an incentive to deviate from mixed strategy  $x_I(\lambda)$  to any strategy for the strategy of the given entrant  $a_E^*(x_I^H)$ . Hence,  $u_I(x_I^H, x_E^H, H) = u_I(x_I^L, x_E^\lambda, H)$  should be hold. Otherwise, assume that  $u_I(x_I^H, x_E^H, H) > u_I(x_I^L, x_E^\lambda, H)$ . Then,

$$\begin{aligned} u_I(x_I^H, a_E^*(\cdot), H) &= u_I(x_I^H, x_E^H, H) \\ &> \lambda u_I(x_I^H, x_E^H, H) + (1 - \lambda) u_I(x_I^L, x_E^\lambda, H) \\ &= u_I(x_I(\lambda), a_E^*(\cdot), H) \end{aligned}$$

so that the incumbent has an incentive to deviate from mixed strategy  $x_I(\lambda)$  to pure strategy  $x_I^H$ . Assume that  $u_I(x_I^H, x_E^H, H) < u_I(x_I^L, x_E^\lambda, H)$ . Similarly, in this case, the incumbent has an

incentive to deviate from mixed strategy  $x_I(\lambda)$  to pure strategy  $x_I^L$ . Hence, the mixed strategy of the equilibrium of the incumbent  $x_I(\lambda)$  satisfies  $u_I(x_I^H, x_E^H, H) = u_I(x_I^L, x_E^\lambda, H)$ , and solving this equation yields  $\lambda$  in the equilibrium. The results can be summarized as the following proposition.

**Proposition 5.1.** *The following three cases occur depending on the conditions in which  $u_I(x_I^H, x_E^H, H)$  is greater than or less than  $u_I(x_I^L, x_E^L, H)$  and  $u_I(x_I^L, x_E^M, H)$ .*

**Case (a)**  $u_I(x_I^H, x_E^H, H) \geq u_I(x_I^L, x_E^L, H)$  if and only if there exists hybrid equilibrium (21) for  $\lambda = 1$ , which is identical to separating equilibrium (15),

**Case (b)**  $u_I(x_I^L, x_E^M, H) < u_I(x_I^H, x_E^H, H) < u_I(x_I^L, x_E^L, H)$  if and only if there exists hybrid equilibrium (21) in which  $\lambda$  satisfies  $u_I(x_I^H, x_E^H, H) = u_I(x_I^L, x_E^\lambda, H)$ , and

**Case (c)**  $u_I(x_I^H, x_E^H, H) \leq u_I(x_I^L, x_E^M, H)$  if and only if there exists hybrid equilibrium (21) for  $\lambda = 0$ , which is identical to pooling equilibrium (16).

## 6 Values of the Incumbents and Comparative Statics

### 6.1 Values in an Equilibrium

In this subsection, the distortion of the values of firms by the presence of asymmetric information is examined. The gains and losses of the values for both types of incumbent and entrant are compared with the case of complete information.

In Case (a), separating equilibrium (15) exists. The value of the incumbent for each demand level  $\theta = H, L$  is given by  $u_I(x_I^\theta, x_E^\theta, \theta)$ . The values of the entrant for demand levels  $\theta = H$  and  $\theta = L$  are given by  $u_E^*(1)$  and  $u_E^*(0)$ , respectively. The entrant for any demand level is completely informed by signaling in this case, so that none of the firms have gain or loss compared with the case of complete information.

In Case (b), there exists hybrid equilibrium (21). For the high demand  $\theta = H$ , the value of the incumbent is  $u_I(x_I(\lambda), a_E^*(\cdot), H)$ . Since  $\lambda$  is set as  $u_I(x_I^H, x_E^H, H) = u_I(x_I^L, x_E^\lambda, H)$ ,  $u_I(x_I(\lambda), a_E^*(\cdot), H)$  is equal to  $u_I(x_I^H, x_E^H, H)$  for any  $\lambda$ . This is derived as follows:

$$\begin{aligned} u_I(x_I(\lambda), a_E^*(\cdot), H) &= \lambda u_I(x_I^H, x_E^H, H) + (1 - \lambda) u_I(x_I^L, x_E^\lambda, H) \\ &= \lambda u_I(x_I^H, x_E^H, H) + (1 - \lambda) u_I(x_I^H, x_E^H, H) \\ &= u_I(x_I^H, x_E^H, H). \end{aligned}$$

Hence, the *ex ante* expected value of the high-demand incumbent is  $u_I(x_I^H, x_E^H, H)$ , and no loss exists compared with the case of complete information. In contrast, the low-demand incumbent losses are  $u_I(x_I^L, x_E^L, L) - u_I(x_I^L, x_E^\lambda, L)$  compared with the case of complete information. The reason of the loss of the low-demand incumbent is explained as follows: the entrant observing that the incumbent invests at  $x_I^L$  invests at  $x_E^\lambda$  in (21), which is earlier than  $x_E^L$  at complete information, and this decreases the value of the low-demand incumbent. Since, the entrant also losses the value by distorting the optimal timing of the exercise of the option under both levels of demand. Consequently, the values of all firms in Case (b) are less than or equal to the values in the case of complete information.

Note that the high-demand incumbent has no loss for both the *ex ante* value and the *ex post* value, as compared to the case of complete information. The mixed strategy realizes trigger  $x_I^H$  with probability  $\lambda$  and trigger  $x_I^L$  with probability  $1 - \lambda$ . Given the realization of the timing at  $x_I^H$ , the entrant invests in the project at  $x_E^H$ , so that the *ex post* value of the high-demand incumbent is  $u_I(x_I^H, x_E^H, H)$ , which is identical to the value in complete information. If the realization of the timing is  $x_I^L$ , then the entrant invests at  $x_E^\lambda$ , and the *ex post* value of the high-demand incumbent is  $u_I(x_I^L, x_E^\lambda, H)$ . Since  $u_I(x_I^H, x_E^H, H) = u_I(x_I^L, x_E^\lambda, H)$ , the value is also same as that in complete information.

Finally, I consider Case (c), in which pooling equilibrium (16) exists. The values of incum-

bents under high and low demand are given by  $u_I(x_I^L, x_E^M, H)$  and  $u_I(x_I^L, x_E^M, L)$ , respectively. Since  $u_I(x_I^L, x_E^M, H) \geq u_I(x_I^H, x_E^H, H)$  holds in Case (c), the high-demand incumbent gains a positive value for  $u_I(x_I^L, x_E^M, H) - u_I(x_I^H, x_E^H, H)$ , as compared to complete information by mimicking the low-demand incumbent and by letting the entrant delay investment. In contrast, the low-demand incumbent losses are  $u_I(x_I^L, x_E^L, L) - u_I(x_I^L, x_E^M, L)$ , as compared with the case of complete information, because the entrant puts the entrance ahead  $x_E^M$  from  $x_E^L$ . The entrant losses the value by distorting the optimal timing for both demand levels, similarly to Case (b). Only the high-demand incumbent gains at (16), where the low-demand incumbent and the entrant are harmed by the strategic behavior of the incumbent under the high demand.

## 6.2 Comparative Statics

In this subsection, influences of various factors on strategic behavior of the incumbent and on the value of players are examined.

First, the behavior of the incumbent under high-demand depends on marginal profit in duopoly of the high-demand incumbent  $\pi_{I2}^H$ . This condition is obtained by solving (17) and (18) to  $\pi_{I2}^H$  as follows. Define  $\xi(\pi_{E2})$  by

$$\xi(\pi_{E2}) = \frac{(\pi_{I1}^H)^\beta - (\pi_{I1}^L)^\beta \phi}{(\pi_{E2}^H)^{\beta-1} - (\pi_{E2})^{\beta-1}}.$$

where

$$\phi = \frac{\beta\pi_{I1}^H - (\beta-1)\pi_{I1}^L}{\pi_{I1}^L}. \quad (22)$$

Proposition 5.1 shows a condition for  $\pi_{I2}^H$  where each type of assessments becomes an equilibrium.

**Proposition 6.1. Case (a)** *Separating equilibrium (15) exists if and only if*

$$\pi_{I2}^H \geq \pi_{I1}^H - \frac{\xi(\pi_{E2}^L)}{\beta} \left( \frac{K_E}{K_I} \right)^{\beta-1},$$

**Case (b)** *hybrid equilibrium (21) exists if and only if*

$$\pi_{I1}^H - \frac{\xi(\pi_{E2}^L)}{\beta} \left( \frac{K_E}{K_I} \right)^{\beta-1} > \pi_{I2}^H > \pi_{I1}^H - \frac{\xi(\pi_{E2}^M)}{\beta} \left( \frac{K_E}{K_I} \right)^{\beta-1},$$

and

**Case (c)** *pooling equilibrium (16) exists if and only if*

$$\pi_{I2}^H \leq \pi_{I1}^H - \frac{\xi(\pi_{E2}^M)}{\beta} \left( \frac{K_E}{K_I} \right)^{\beta-1}.$$

**Proof.** *See Appendix.*

Proposition 6.1 states that larger marginal profit in duopoly under high demand ensures that the high-demand incumbent does not act strategically and invests at the optimal timing for the entrant's timing is fixed. This is because the high-demand incumbent has less incentive to prevent earlier investment of the entrant. In contrast, less marginal profit in duopoly under high demand makes the high-demand incumbent to hide information and take advantage of the monopoly for a longer time. Hence, in this case, strategically, the high-demand incumbent invests in the project at the optimal timing of the low-demand incumbent in order to hide the information. In the mid-range of marginal profit in duopoly, the high-demand incumbent uses a mixed strategy.

Second, the behavior of the incumbent under high-demand depends on the investment costs of the incumbent and the entrant,  $K_I$  and  $K_E$ . Proposition 6.2 is obtained by solving the equations in 6.1 to  $\frac{K_E}{K_I}$ .

**Proposition 6.2.** **Case (a)** *a separating equilibrium (15) exists if and only if*

$$\frac{K_E}{K_I} \geq \left\{ \frac{\beta}{\xi(\pi_{E2}^L)} (\pi_{I1}^H - \pi_{I2}^H) \right\}^{\frac{1}{\beta-1}},$$

**Case (b)** *a hybrid equilibrium (21) exists if and only if*

$$\left\{ \frac{\beta}{\xi(\pi_{E2}^L)} (\pi_{I1}^H - \pi_{I2}^H) \right\}^{\frac{1}{\beta-1}} > \frac{K_E}{K_I} > \left\{ \frac{\beta}{\xi(\pi_{E2}^M)} (\pi_{I1}^H - \pi_{I2}^H) \right\}^{\frac{1}{\beta-1}},$$



and

**Case (c)** *the pooling equilibrium (16) exists if and only if*

$$\frac{K_E}{K_I} \geq \left\{ \frac{\beta}{\xi(\pi_{E2}^M)} (\pi_{I1}^H - \pi_{I2}^H) \right\}^{\frac{1}{\beta-1}}.$$

Proposition 6.2 asserts that either a sufficiently lower cost of the incumbent or larger cost of the entrant makes the high-demand incumbent acts truthfully and invests at the optimal timing under complete information. In contrast, under a larger cost of the incumbent or a lower cost of the entrant, the high-demand incumbent has the incentive of the strategic behavior.

## 7 Numerical Examples for Equilibrium Strategies and Values

In this section, we show examples for results of comparative statics for the behavior of the incumbent under the high demand and the values of the incumbent are presented through numerical examples. Parameters in examples are basically set as  $\mu = 0.03$ ,  $r = 0.07$ ,  $p = 0.5$ ,  $\sigma = 0.2$ ,  $x = 0.05$ ,  $\pi_{I1}^H = 12$ ,  $\pi_{I1}^L = 7$ ,  $\pi_{I2}^H = 4$ ,  $\pi_{I2}^L = 4$ ,  $\pi_{E2}^H = 4$ ,  $\pi_{E2}^L = 1$ ,  $K_I = 50$ , and  $K_E = 100$ .

First, relations between values of the incumbent under the high demand  $u_I(\cdot, \cdot, H)$  and marginal profit in duopoly of the high-demand incumbent  $\pi_{I2}^H$  are examined. Figure 1 illustrates the values  $u_I(x_I^H, x_E^H, H)$ ,  $u_I(x_I^L, x_E^M, H)$ , and  $u_I(x_I^L, x_E^L, H)$ .  $u_I(x_I^H, x_E^H, H)$  is greater than  $u_I(x_I^L, x_E^L, H)$  for  $\pi_{I2}^H \geq 8.0$ . As explained in Proposition 6.1, the high-demand incumbent does not deviate the optimal timing of the investment truthfully for  $\pi_{I2}^H \geq 8.0$ , because marginal profit in duopoly of the incumbent is sufficiently large and the incumbent does not have an incentive to make the delay the entrant's investment. Hence, the high-demand incumbent invests in the project at the optimal timing under complete information, and reveals his information truthfully. In contrast, for  $\pi_{I2}^H \leq 2.9$ ,  $u_I(x_I^H, x_E^H, H)$  is less than  $u_I(x_I^L, x_E^M, H)$ . In this range, the

high-demand incumbent invests at the optimal timing of the low demand to hide information about high demand. For  $2.9 \leq \pi_{I2}^H \leq 8.0$ ,  $u_I(x_I^L, x_E^M, H) \leq u_I(x_I^H, x_E^H, H) \leq u_I(x_I^L, x_E^L, H)$ , the incumbent uses a mixed strategy as (21). In this interval, the value of the incumbent is the same as  $u_I(x_I^H, x_E^H, H)$  because the mixed strategy should satisfy the condition  $u_I(x_I^H, x_E^H, H) = u_I(x_I^L, x_E^L, H)$ . Therefore, the value of the high-demand incumbent in the equilibrium strategy is  $u_I(x_I^H, x_E^H, H)$  for  $\pi_{I2}^H \leq 2.9$  and is  $u_I(x_I^L, x_E^M, H)$  for  $\pi_{I2}^H \geq 2.9$ .

Figure 2 illustrates the probability  $\lambda$  that the high-demand incumbent invests at the optimal timing for the high demand in the equilibrium strategy versus marginal profit in duopoly of the high-demand incumbent  $\pi_{I2}^H$ . For  $\pi_{I2}^H \leq 2.9$ , the pooling equilibrium (16) exists so that  $\lambda = 0$ , while for  $\pi_{I2}^H \geq 8.0$ , a separating equilibrium (15) exists so that  $\lambda = 1$ . For  $2.9 < \pi_{I2}^H < 8.0$ , the incumbent uses a completely mixed strategy, and  $\lambda$  has a positive value, which increases in  $\pi_{I2}^H$ .

Third, the effect of volatility is examined. Figure 3 illustrates relation between the values of the high-demand incumbent and the volatility  $\sigma$ . If the volatility is small, the incumbent invests truthfully, whereas if the volatility is large, the incumbent invests strategically. If the volatility is moderate, the incumbent uses a mixed strategy.

Fourth, relation between values of the incumbent and an investment cost of the incumbent is investigated. Figure 4 depicts relation between values of the high-demand incumbent and its cost of the investment. As explained in Proposition 6.2, the values decrease non-linearly in an investment cost of the incumbent. If the cost is small, a separating equilibrium (15) exists, whereas if the cost is large, a separating equilibrium (16) exists. If the cost is moderate, the incumbent uses a complete mixed strategy, in which there exists the hybrid equilibrium (21) for some  $0 < \lambda < 1$ .

Finally, relation between values of the incumbent and investment cost of the entrant is investigated. Interestingly, the value of the incumbent is affected not only by parameters of

the incumbent, but also by parameters of the rival due to strategic interaction. Smaller cost of the entrant pushes forward the timing of the entrant, reduces the value of the incumbent. Figure 5 depicts relation between values of the high-demand incumbent and cost of the entrant. As shown shown in Proposition 6.2, if the cost of the entrant is large, then the timing of the investment of the entrant is late. Then an effect of the timing of the entrant on the value of the incumbent is negligible, the high-demand incumbent invests at the optimal timing under complete information truthfully. On the other hand, the incumbent invests strategically for the case in which the cost of the entrant is small. For a moderate interval of the cost of the entrant, the incumbent uses a mixed strategy.

## 8 Conclusion

The present paper examines an investment game for an incumbent and an entrant for optimal timing of the investment in which only the incumbent has the information of the demand and the entrant predicts the demand by observing the investment timing of the incumbent. Signaling effect is considered and the concept of a perfect Bayesian equilibrium is used as a solution concept. A condition in which the incumbent under the high demand invests strategically in the equilibrium is characterized. A condition in which the incumbent to use a mixed strategy in the equilibrium is also demonstrated.

If marginal profit in duopoly for the high-demand incumbent is small, then the incumbent invests strategically, whereas the incumbent invests truthfully if marginal profit in duopoly is sufficiently large. The incumbent also invests strategically, if the volatility or the cost of the incumbent is large, or if the cost of the entrant is small.

This is the first study of signaling model for an investment game under real option approach in my knowledge and an extension of the model would be interesting in future research. Preemptive

behavior should be considered by eliminating the assumption in which the incumbent is the leader and the entrant is the follower. Other stochastic processes could also be considered in order to extend the model.

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## Appendix

**Proof of Proposition 3.1:** This proposition is derived by a standard calculation of the first hitting time (see for example [2], pp. 315–316). Let  $\hat{t}$  be the first hitting time at which  $X_s$  reaches a fixed threshold  $\hat{x}$ , where  $X_0 = x$ . Dixit and Pindyck (1994, pp. 315–316) reported that

$$E[e^{-r\hat{t}}] = \left(\frac{x}{\hat{x}}\right)^\beta \quad (23)$$

and

$$E\left[\int_0^T X_s e^{-rs} ds\right] = \frac{x}{r-\mu} - \frac{\hat{x}}{r-\mu} \left(\frac{x}{\hat{x}}\right)^\beta \quad (24)$$

where  $\beta$  is given by (2). (24) implies that

$$E\left[\int_T^{+\infty} X_s e^{-rs} ds\right] = E\left[\int_0^{+\infty} X_s e^{-rs} ds\right] - E\left[\int_0^T X_s e^{-rs} ds\right] = \frac{\hat{x}}{r-\mu} \left(\frac{x}{\hat{x}}\right)^\beta \quad (25)$$

First, I consider  $v_I(x_I, \theta)$ . If  $x > x_I$ , then the incumbent immediately invests, i.e.,  $t_I = t$ .

Hence, by the Markov property of the geometric Brownian motion, I have

$$\begin{aligned}
v_I(x_I, \theta) &= E\left[\int_{t_I}^{\infty} e^{-r(s-t)} \pi_{I1}^{\theta} X_s ds - e^{-r(t_I-t)} K_I | X_t = x\right] \\
&= E\left[\int_t^{\infty} e^{-r(s-t)} \pi_{I1}^{\theta} X_s ds - K_I | X_t = x\right] \\
&= E\left[\int_0^{\infty} e^{-rs} \pi_{I1}^{\theta} X_s ds | X_0 = x\right] - K_I \\
&= \frac{\pi_{I1}^{\theta}}{r - \mu} x - K_I.
\end{aligned}$$

If  $x \leq x_I$ , then  $t_I$  is the first hitting time of the stochastic process reaches a fixed threshold  $x_I$ . Then, according to the Markov property of the geometric Brownian motion,

$$\begin{aligned}
v_I(x_I, \theta) &= E\left[\int_{t_I}^{\infty} e^{-r(s-t)} \pi_{I1}^{\theta} X_s ds - e^{-r(t_I-t)} K_I | X_t = x\right] \\
&= E\left[\int_{t_I}^{\infty} e^{-rs} \pi_{I1}^{\theta} X_s ds - e^{-rt_I} K_I | X_0 = x\right] \\
&= E\left[\int_{t_I}^{\infty} e^{-rs} \pi_{I1}^{\theta} X_s ds | X_0 = x\right] - K_I E[-e^{-rt_I} | X_0 = x].
\end{aligned}$$

(23) and (25) imply that  $v_I(x_I, \theta) = \left(\frac{\pi_{I1}^{\theta}}{r - \mu} x_I - K_I\right) \left(\frac{x}{x_I}\right)^{\beta}$

Similarly,  $w_I(x_E, \theta)$  for  $x < x_E$  is

$$\begin{aligned}
w_I(x_E, \theta) &= E\left[\int_{t_E}^{\infty} e^{-r(s-t)} (\pi_{I1}^{\theta} - \pi_{I2}^{\theta}) X_s ds | X_t = x\right] \\
&= E\left[\int_{t_E}^{\infty} e^{-rs} (\pi_{I1}^{\theta} - \pi_{I2}^{\theta}) X_s ds | X_0 = x\right] \\
&= \frac{\pi_{I1}^{\theta} - \pi_{I2}^{\theta}}{r - \mu} x_E \left(\frac{x}{x_E}\right)^{\beta}.
\end{aligned}$$

□

### Proof of Proposition 3.1:

This proposition is derived by solving (17) and (18) for  $\pi_{I2}^H$ . In this proof, the condition of Case (a) is shown to be obtainable by solving (17). The conditions of Case (b) and Case (c) are obtained in a similar manner.

According to (4), the inequality (17) is expressed by

$$v_I(x_I^H, H) - w_I(x_E^H, H) \geq v_I(x_I^L, H) - w_I(x_E^L, H),$$

which can be rewritten as

$$v_I(x_I^H, H) - v_I(x_I^L, H) \geq w_I(x_E^H, H) - w_I(x_E^L, H). \quad (26)$$

Then, (5) implies

$$v_I(x_I^H, H) - v_I(x_I^L, H) = \left( \frac{\pi_{I1}^H}{r - \mu} x_I^H - K_I \right) \left( \frac{x}{x_I^H} \right)^\beta - \left( \frac{\pi_{I1}^H}{r - \mu} x_I^L - K_I \right) \left( \frac{x}{x_I^L} \right)^\beta.$$

Substituting (9) into the above expression, I obtain

$$v_I(x_I^H, H) - v_I(x_I^L, H) = \beta^{-\beta} (\beta - 1)^{\beta-1} x^\beta K_I^{1-\beta} (r - \mu)^{-\beta} \{ (\pi_{I1}^H)^\beta - \phi (\pi_{I1}^L)^\beta \}, \quad (27)$$

where  $\phi$  is defined by (22). (6) also implies that

$$w_I(x_E^H, H) - w_I(x_E^L, H) = \frac{\pi_{I1}^H - \pi_{I2}^H}{r - \mu} x_E^H \left( \frac{x}{x_E^H} \right)^\beta - \frac{\pi_{I1}^H - \pi_{I2}^H}{r - \mu} x_E^L \left( \frac{x}{x_E^L} \right)^\beta \quad (28)$$

By (1),  $x_E^H$  and  $x_E^L$  are given by

$$x_E^H = x_E^*(1) = \frac{\beta}{\beta-1} \frac{r-\mu}{\pi_{E2}^H} K_E, \quad x_E^L = x_E^*(0) = \frac{\beta}{\beta-1} \frac{r-\mu}{\pi_{E2}^L} K_E.$$

Hence, by substituting these expressions into (29), I obtain

$$w_I(x_E^H, H) - w_I(x_E^L, H) = \beta^{1-\beta} (\beta-1)^{\beta-1} x^\beta K_E^{1-\beta} (r-\mu)^{-\beta} (\pi_{I1}^H - \pi_{I2}^H) \{ (\pi_{I1}^H)^{\beta-1} - (\pi_{I1}^L)^{\beta-1} \}. \quad (29)$$

According to (27) and (29), inequality (26) implies that

$$K_I^{1-\beta} \{ (\pi_{I1}^H)^\beta - \phi (\pi_{I1}^L)^\beta \} \geq \beta K_E^{1-\beta} (\pi_{I1}^H - \pi_{I2}^H) \{ (\pi_{I1}^H)^{\beta-1} - (\pi_{I1}^L)^{\beta-1} \}.$$

Consequently, this yields

$$\pi_{I2}^H \geq \pi_{I1}^H - \frac{1}{\beta} \left\{ \frac{(\pi_{I1}^H)^\beta - \phi (\pi_{I1}^L)^\beta}{(\pi_{I1}^H)^{\beta-1} - (\pi_{I1}^L)^{\beta-1}} \right\} \left( \frac{K_E}{K_I} \right)^{\beta-1},$$

which completes the proof.  $\square$

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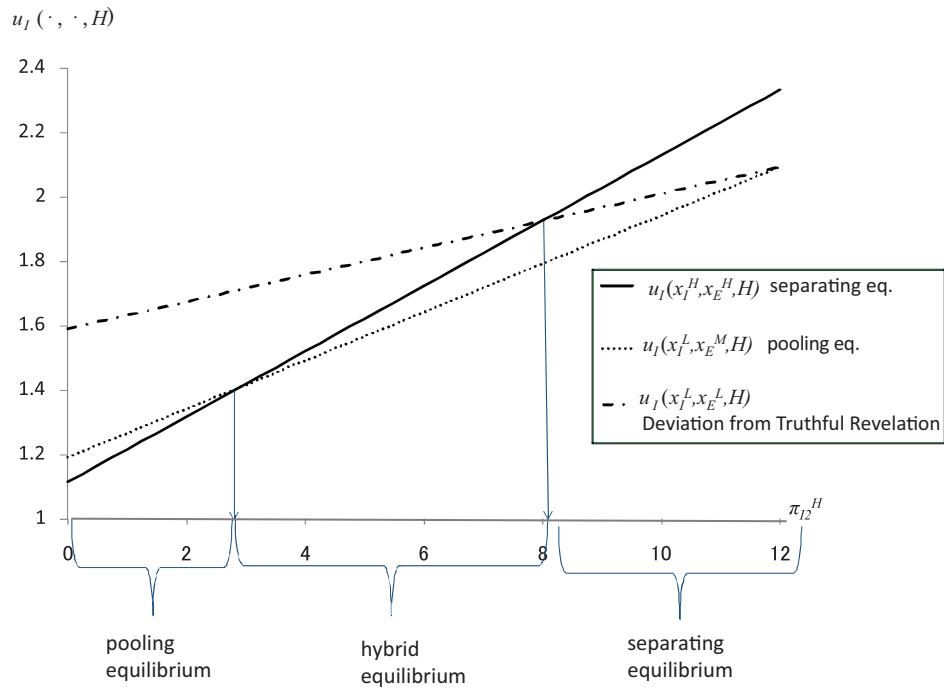


Figure 1: Values of the high-demand incumbent  $u_I(\cdot, \cdot, H)$  and the duopoly profit of the high-demand incumbent  $\pi_{I2}^H$

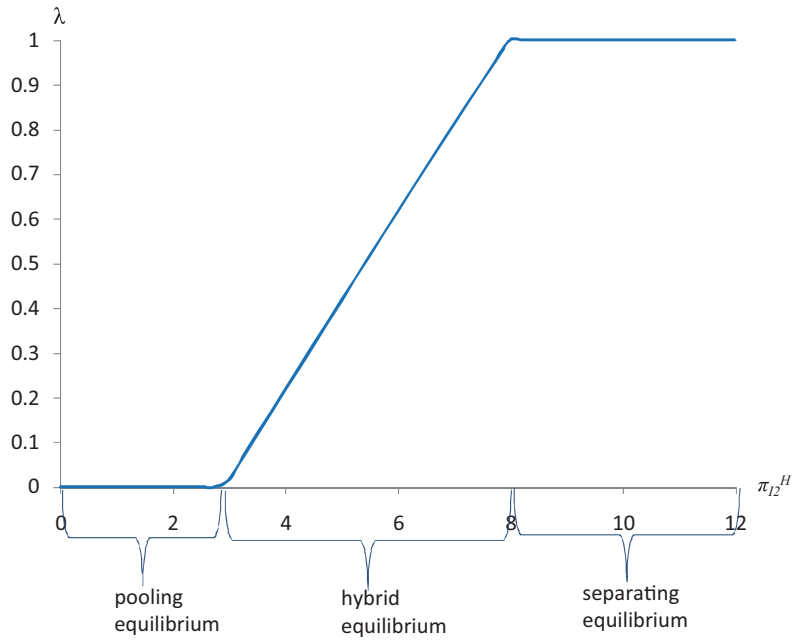


Figure 2:  $\lambda$  (probability of the investment at  $x_I^H$ ) and the duopoly profit of the high-demand incumbent  $\pi_{I2}^H$

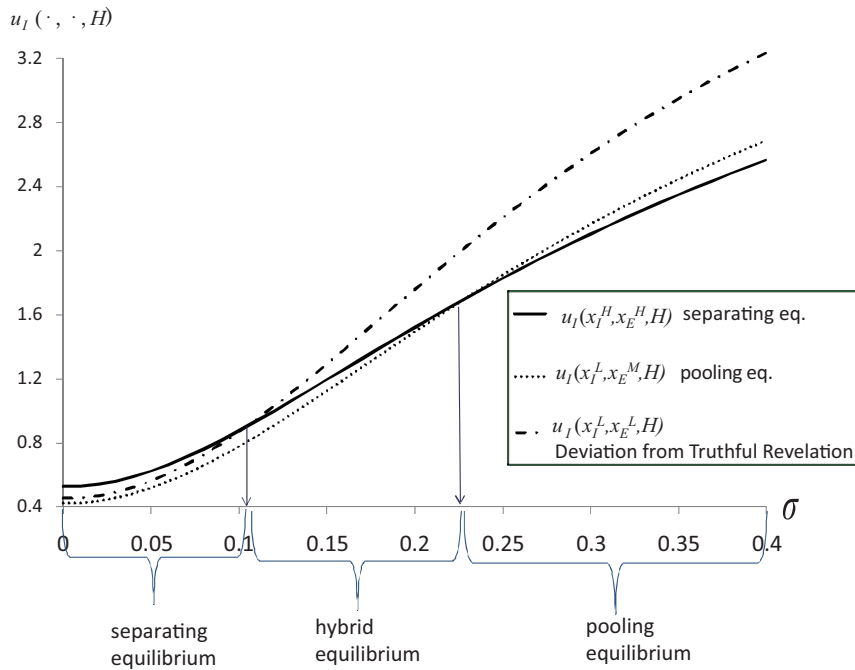


Figure 3: Values of the high-demand  $u_I(\cdot, \cdot, H)$  and volatility  $\sigma$

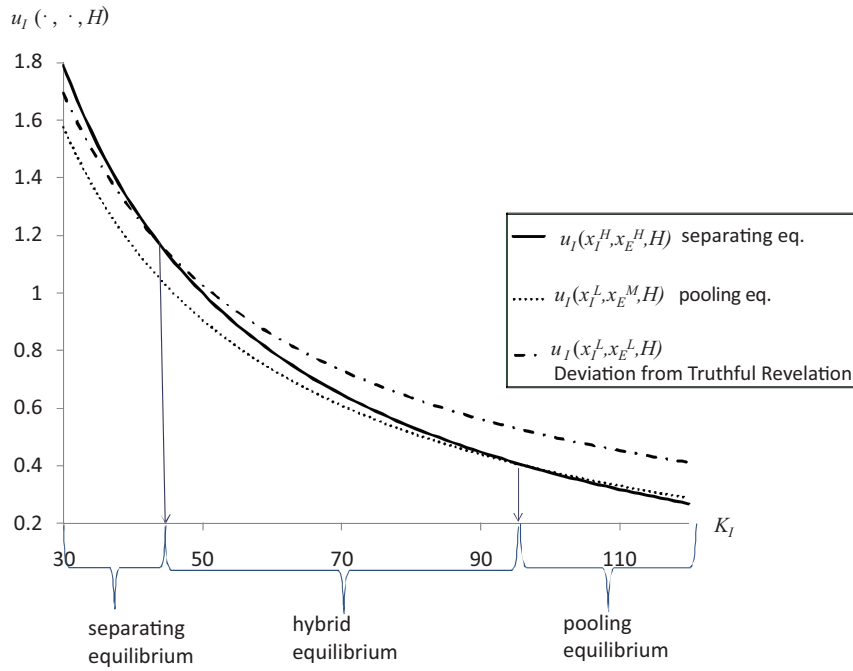


Figure 4: Values of the high-demand incumbent  $u_I(\cdot, \cdot, H)$  and the investment cost of the high-demand incumbent  $K_I$

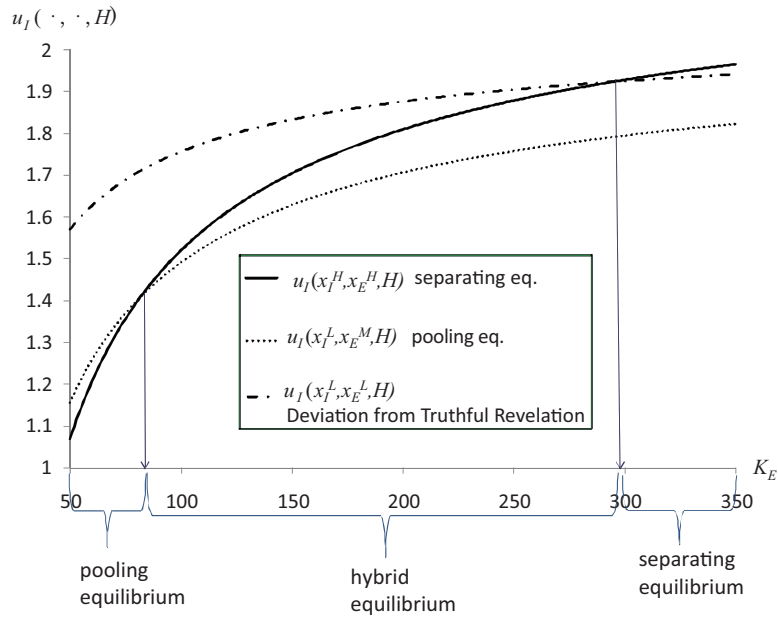


Figure 5: Values of the high-demand incumbent  $u_I(\cdot, \cdot, H)$  and the investment cost of the entrant  $K_E$